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On Superimposed Recurrent Cycles

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On Superimposed Recurrent Cycles*

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Following a proposal of Lawrence R. Klein, we show that time series exhibiting irregular fluctuations can be described by difference equations with several pairs of undamped complex roots. This evidence contrasts with the theory of dying cycles, which are kept alive with random shocks.

1. Introduction

In the course of time the study of business cycles developed from the theories of self-sustaining cycles into the theories of cyclical response to exogenous shocks, mainly of monetary origine. The older theories from the thirties and fourties had the common feature of being based on an endogenous dynamism, resulting from the interaction between the multiplier and some form of acceleration principle. Especially according to the range of values in which the parameters of the investment function were chosen, these models produced explosive, damped or constant cycles.

After World War II there was a rise of interest in the case of explosive fluctuations, in the meantime neglecting expectational and other important aspects of economic reality. Afterwards it is quite understandable that these theories of endogenous instability induced strong reactions, by monetarism but also by macroeconometric modelbuilding.** But after the disappointing experience of the early seventies, there

*) The authors thank Th. van de Klundert for his comment on an earlier version of this paper.

**) About forty relevant articles from the period 1933-1963 have been republished in Readings in Business Cycles (1966).

arised a new chain of theories, now based on the hope to reconcile business cycles with the postulates of microeconomic (competitive) theory. Initially this approach relied upon random monetary and related shocks that were held to be responsable for cyclical fluctuations that after the stable sixties still proved to be well alive. Later on rational expectationists tried to introduce more identifiable exogenous factors into this view, that actually was as one-sided as the older theories of predominantly endogenous cycles.

In an extensive review of the literature on business cycles, Zarnowitz (1985) confronts these theories with what he calls the 'stylized facts'. These indeed tell us that the observed fluctuations do not resemble the deterministic wave motions which sometimes arise in the natural sciences. But on the other hand it appears that cycles are persistent, lasting long enough to permit the development of cumulative movements, both in the downward and in the upward direction. The observed movements moreover have in common that they show up in many ways, not only in macroeconomic variables, but also in spatial and sectoral magnitudes. For the United States in particular, it has been well established that the mean duration of business cycles remained approximately stable at four years, in which during the last four decades expansions covered about three years and contractions about one year.

At the end of his survey Zarnowitz pleads for a synthesis between the theories of self-sustaining cycles and the new mainstream literature on rational expectations models. We agree with this, but we think it is a pity that he does not incorporate the lessons which can be learned from macroeconometric modelbuilding into his proposal. Some of these lessons are told in Klein (1983). And starting from these we will present here a methodological device to give some ground to the desired synthesis.

Central to this is the hypothesis that fluctuations in economic time series are essentially recurrent. As Klein indicates, this is supported by spectral and autoregressive analysis of stochastic simulations with nonlinear models. Actually we show that indeed it is possible to combine enough cosine functions to approximate any given time series. This is Klein's proposal, which will be described in section 2. In section 3 we present a numerical example, which is meant to illustrate that the framework proposed can be given empirical content, namely by combining the theory of self-sustaining cycles with the hypothesis of identifiable exogenous shocks. In section 4 the method is applied to a time series of real life: the rate of capacity utilization in U.S. manufacturing. The epilogue gives some suggestions for other applications of the method described.

2. Klein's proposal

Some years ago Klein (1983) presented a method of cyclical analysis, which was basically meant to be applied to simulated time series, but in our opinion can be used for analysing actual series as well.

As a starting point, consider the second order difference equation

$$y_t + \rho y_{t-1} + y_{t-2} = 0.$$

Notice that the coefficient of y_{t-2} has been put at unity, so that the constant term of the corresponding characteristic equation also assumes the value one:

$$\lambda^2 + \rho\lambda + 1 = 0.$$

The roots of this equation are conjugate complex if $\rho^2 < 4$. For, if this inequality holds, the sign of the discriminant in

$$\frac{-\rho \pm \sqrt{\rho^2-4}}{2}$$

is negative, so that the roots have the form

$$a \pm ib.$$

The equivalent trigonometric form for complex numbers is:

$$a = r\cos\theta \; , \; b = r\sin\theta, \text{ with modulus } r = \sqrt{a^2+b^2}.$$

For our equation the modulus of the roots

$$\sqrt{\frac{\rho^2+4-\rho^2}{4}} = 1,$$

so that the solution will show a recurrent cycle. Then, the angle of oscillation can be calculated at:

$$\theta = \cos^{-1} a = \cos^{-1}(-\tfrac{1}{2}\rho).$$

Next, consider the time series, y_t. For this, Klein recommends to fit the equation

$$y_t + y_{t-2} = -\rho y_{t-1} + u_t,$$

by minimizing the residual error in the square. This procedure restricts the coefficient of y_{t-2} to be one, thus insuring no dampening. The estimated parameter then can be used to estimate θ in

$$\hat{\theta} = \cos^{-1}(-\tfrac{1}{2}\hat{\rho}).$$

In this way the angle of oscillation of the best-fitting undamped sinusoid to the data provides an estimate of the periodicity of the series. If for instance, the estimated value of $\hat{\rho} = 1$, then $\hat{\theta} = 120°$, or if $\hat{\rho} = 0$, then $\hat{\theta} = 90°$, or if $\hat{\rho} = -1$, then $\hat{\theta} = 60°$. From this it follows that we have found cycles of respectively 3, 4 and 6 periods.

As Klein shows, this method can easily be extended to the more general case of superimposed cycles. In the case of the fourth order equation

$$y_t + \rho_1 y_{t-1} + \rho_2 y_{t-2} + \rho_3 y_{t-3} + y_{t-4} = 0,$$

the characteristic equation is

$$\lambda^4 + \rho_1\lambda^3 + \rho_2\lambda^2 + \rho_3\lambda + 1 = 0.$$

The roots of this equation will appear as two pairs of conjugate complex roots, if the following factoring is possible:

$$(\lambda - a \pm ib)(\lambda - c \pm id) = 0.$$

By multiplying out, we find

$$\lambda^4 - (2a+2c)\lambda^3 + (a^2+b^2+c^2+d^2+4ac)\lambda^2 - \{2c(a^2+b^2) + 2a(c^2+d^2)\}\lambda + (a^2+b^2)(c^2+d^2) = 0.$$

For one pair of conjugate complex roots we already know that no dampening is guaranteed if

$$a^2 + b^2 = 1, \text{ and/or } c^2 + d^2 = 1.$$

Using these conditions, the characteristic equation can be simplified to

$$\lambda^4 - (2a + 2c)\lambda^3 + (2 + 4ac)\lambda^2 - (2a + 2c)\lambda + 1 = 0.$$

Notice that $\rho_1 = \rho_3$, so that it is clear that in the underlying case it is recommended to regress

$$y_t + y_{t-4} = -\rho_1(y_{t-1} + y_{t-3}) - \rho_2 y_{t-2} + u_t.$$

Then, analogous to the second order case, the estimated parameters can be used to estimate the angles of oscillation in

$$\hat{\theta}_1 = \cos^{-1}\hat{a} \text{ and } \hat{\theta}_2 = \cos^{-1}\hat{c}.$$

If for instance it has been found that $\hat{\rho}_1 = 1$ and $\hat{\rho}_2 = 2$, then $\hat{a} = -0.5$ and $\hat{c} = 0$, so that $\hat{\theta}_1 = 120°$ and $\hat{\theta}_2 = 90°$.

It is worthwile to note that the time series belonging to this example shows a cycle, which can be calculated at 12 periods. We shall call this the dominant cycle and the other ones the underlying cycles. It is easy to see that the periodicity of the dominant cycle equals the least common multiple of the periodicities of the underlying cycles. (This will be illustrated in the next section.)

As Klein indicates, the procedure outlined above can be extended as far as one wants. In the case of a sixth order equation, we regress

$$y_t + y_{t-6} = -\rho_1(y_{t-1} + y_{t-5}) - \rho_2(y_{t-2} + y_{t-4}) - \rho_3 y_{t-3} + u_t.$$

And analogously to before the estimated ρ's can be used to estimate the θ's in

$$\hat{\theta}_1 = \cos^{-1}\hat{a},\ \hat{\theta}_2 = \cos^{-1}\hat{c} \text{ and } \hat{\theta}_3 = \cos^{-1}\hat{e},$$

where a, c and e are calculated from*

$$\rho_1 = -2a - 2c - 2e,$$
$$\rho_2 = 3 + 4ac + 4ae + 4ce,$$
$$\rho_3 = -4a - 4c - 4e - 8ace.$$

If for instance, the estimated parameters are

$$\hat{\rho}_1 = -2.146,\ \hat{\rho}_2 = 2.303 \text{ and } \hat{\rho}_3 = -1.843,$$

then

$$\hat{a} = -0.5,\ \hat{c} = 0.707 \text{ and } \hat{e} = 0.866,$$

*) The coefficient k of a $2n^{th}$ order difference equation with undamped complex roots can be calculated by:

$$\rho_k^n = (-1)^k \sum_i^k \begin{bmatrix} n-i \\ \frac{1}{2}(k-i) \end{bmatrix} C_i^n$$

<u>Example</u>: n=3

$$\rho_1 = (-1)^1[2a + 2c + 2e]$$

$$\rho_2 = (-1)^2[3 + 2a2c + 2a2e + 2c2e]$$

$$\rho_3 = (-1)^3[2(2a + 2c + 2e) + 2a2c2e]$$

so that

$$\hat{\theta}_1 = 120°, \ \hat{\theta}_2 = 45° \text{ and } \hat{\theta}_3 = 30°.$$

This is an example with a dominant cycle of 24 periods, being the least common multiple of the three underlying cycles, the first of 3, the second of 8 and the third of 12 periods.

3. A theoretical example

We now will demonstrate how Klein's proposal can be used to combine the theory of self-sustaining cycles with the hypothesis of identifiable exogenous shocks. To this end consider the (fictive) time series that is reproduced by Figure 1.a. (The corresponding figures are given in Appendix 1.)

In order to discover the autoregressive equation that describes the series best, several specifications have been tested. These (OLS) regressions are set out in Table 1. The equations (1) - (8) are unrestricted relationships. From the given statistics it is easy to see that the lower order equations are not very adequate. Moving to a higher order provides us with a better result.

However, this process is clearly not infinite, for the seventh and eigth order equations do not add anything to the satisfying result that is obtained when using a sixth order equation. (At least according to the additional variables' low t-ratio's.) This sixth order equation also reveals that some of the estimated coefficients do not deviate from zero significantly, whereas at the same time it appears that $\rho_2 \approx \rho_4$ and $\rho_6 \approx 1$, thus giving ground to the hypothesis that the sequence can be considered as a recurrent cycle. This is tested by regressing $(y_t + y_{t-6})$ on $(y_{t-2} + y_{t-4})$. The result is equation (9).

As is shown at the bottom of the table, equation (9) presents the best estimation results found so far. The residuals of this equation, defined as the difference between the actual value of $(y_t + y_{t-6})$ in a certain period and its predicted value, are reproduced by Figure 2. From this it is clear that some relatively large residuals can be observed, namely in the periods 14, 24, 32, 38 and 44. This strongly suggests that

Figure 1.a Time series with irregular fluctuations

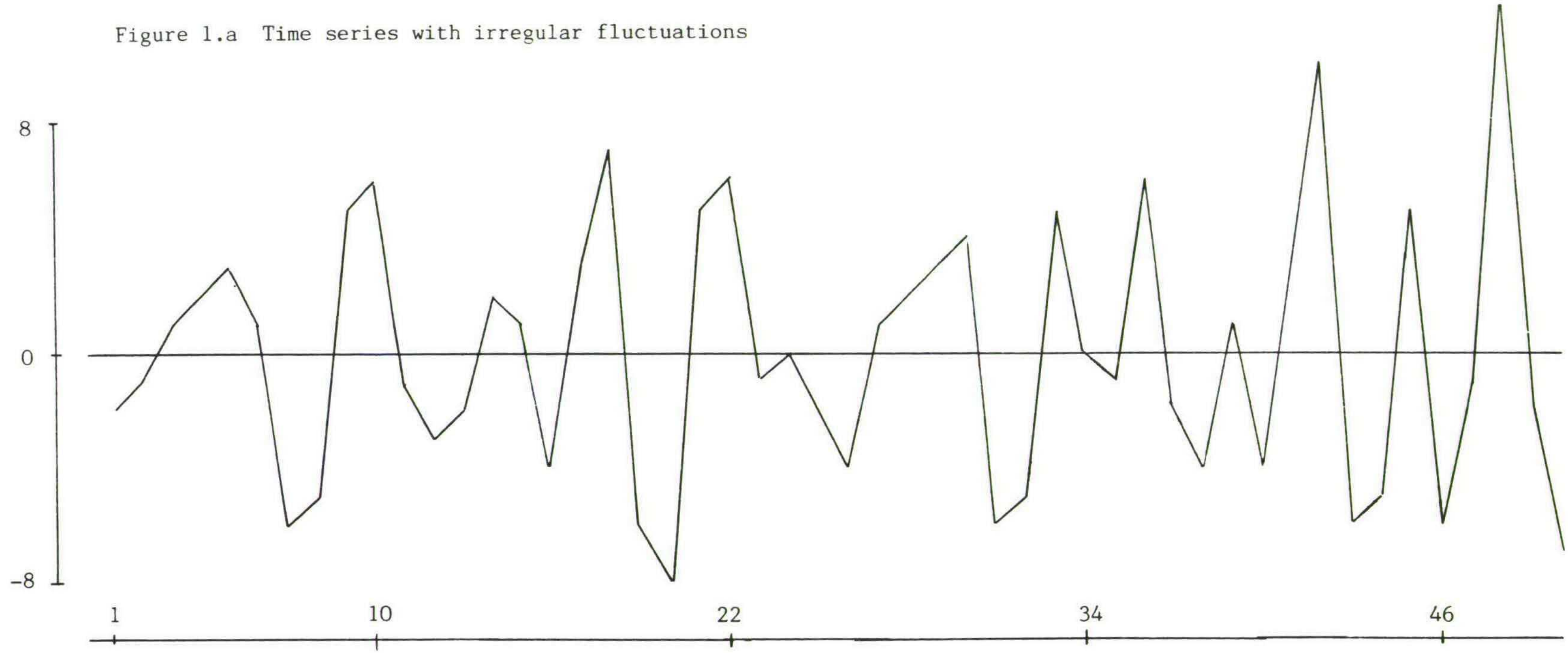


Figure 1.b Time series with dominant cycle of 12 periods

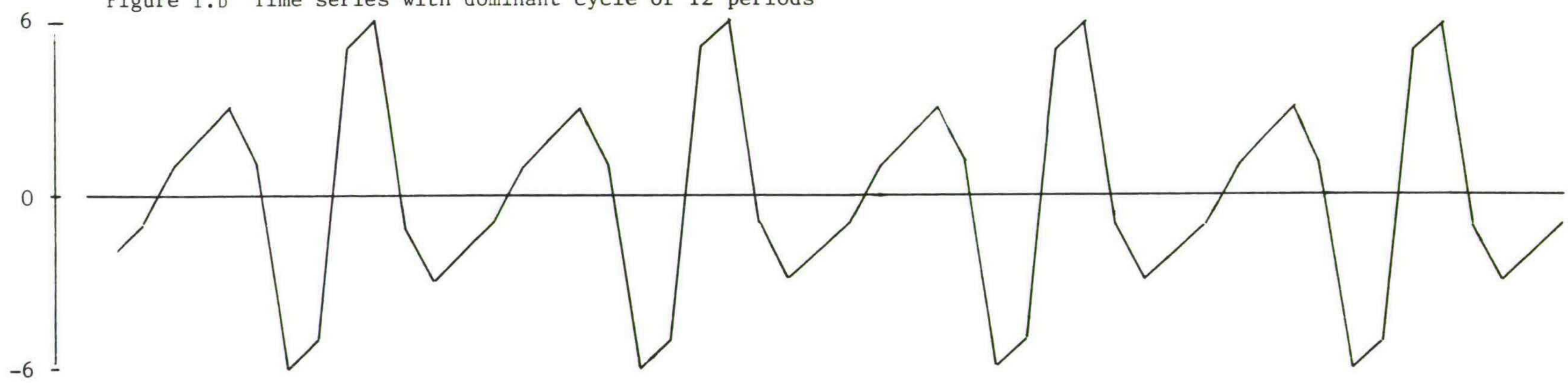


Table 1. Regressions on the fictive time series*

1. $$\underset{}{y_t} = \underset{(0.07)}{0.049} - \underset{(-0.40)}{0.060}\, y_{t-1}$$

$R^2_{d.f.} = 0.0\%$

2. $$y_t = \underset{(0.40)}{0.208} - \underset{(-0.97)}{0.113}\, y_{t-1} - \underset{(-5.70)}{0.664}\, y_{t-2}$$

$R^2_{d.f.} = 39.6\%$

3. $$y_t = \underset{(0.40)}{0.216} - \underset{(-0.42)}{0.064}\, y_{t-1} - \underset{(-5.48)}{0.658}\, y_{t-2} + \underset{(0.53)}{0.086}\, y_{t-3}$$

$R^2_{d.f.} = 38.5\%$

4. $$y_t = \underset{(0.80)}{0.264} + \underset{(0.03)}{0.003}\, y_{t-1} - \underset{(-12.59)}{1.154}\, y_{t-2} + \underset{(0.66)}{0.066}\, y_{t-3} - \underset{(-8.83)}{0.873}\, y_{t-4}$$

$R^2_{d.f.} = 78.2\%$

5. $$y_t = \underset{(0.63)}{0.213} + \underset{(0.04)}{0.006}\, y_{t-1} - \underset{(-12.39)}{1.152}\, y_{t-2} + \underset{(0.36)}{0.077}\, y_{t-3} - \underset{(-8.60)}{0.866}\, y_{t-4} + \underset{(0.06)}{0.011}\, y_{t-5}$$

$R^2_{d.f.} = 78.0\%$

6. $$y_t = \underset{(2.27)}{0.317} - \underset{(-0.44)}{0.029}\, y_{t-1} - \underset{(-29.07)}{1.891}\, y_{t-2} + \underset{(0.44)}{0.038}\, y_{t-3} - \underset{(-22.21)}{1.925}\, y_{t-4} + \underset{(0.37)}{0.026}\, y_{t-5} - \underset{(-13.88)}{0.985}\, y_{t-6}$$

$R^2_{d.f.} = 96.4\%$

7. $$y_t = \underset{(2.64)}{0.405} - \underset{(-1.23)}{0.209}\, y_{t-1} - \underset{(-28.58)}{1.885}\, y_{t-2} - \underset{(-1.01)}{0.331}\, y_{t-3} - \underset{(-21.65)}{1.904}\, y_{t-4} - \underset{(-1.08)}{0.372}\, y_{t-5} - \underset{(-13.47)}{0.968}\, y_{t-6} - \underset{(-1.21)}{0.226}\, y_{t-7}$$

$R^2_{d.f.} = 96.4\%$

8. $$y_t = \underset{(3.15)}{0.537} - \underset{(-1.66)}{0.287}\, y_{t-1} - \underset{(-12.18)}{2.079}\, y_{t-2} - \underset{(-1.41)}{0.468}\, y_{t-3} - \underset{(-7.02)}{2.297}\, y_{t-4} - \underset{(-1.44)}{0.499}\, y_{t-5} - \underset{(-4.06)}{1.396}\, y_{t-6} - \underset{(-1.52)}{0.285}\, y_{t-7} - \underset{(-1.31)}{0.247}\, y_{t-8}$$

$R^2_{d.f.} = 96.5\%$

9. $$y_t = \underset{(2.32)}{0.323} - \underset{(-54.74)}{1.922}\, y_{t-2} - \underset{(-54.74)}{1.922}\, y_{t-4} - y_{t-6}$$

$R^2_{d.f.} = 98.6\%$

10. $$y_t = -2.000\, y_{t-2} - 2.000\, y_{t-4} - y_{t-6} + 3.000\, D_{14} + 3.000\, D_{24} + 3.000\, D_{32} + 3.000\, D_{38} + 3.000\, D_{44}$$

$R^2_{d.f.} = 100\%$ $\theta_1 = 120°$ $\theta_2 = 90°$ $\theta_3 = 60°$

* Numbers between brackets are t-ratio's.

certain shocks, which are both scarce and apparent, have been active. Of course this can be investigated with the aid of dummy variables. Adding these, the result is the last equation of Table 1, from which can be said that it describes the time series perfectly.

According to equation (10) we now are able to conclude that we have identified five exogenous shocks, each equal to 3. At the same time we have discovered that the 'real' autoregressive part of the sequence is given by

$$\text{I} \qquad y_t = -2y_{t-2} - 2y_{t-4} - y_{t-6}$$

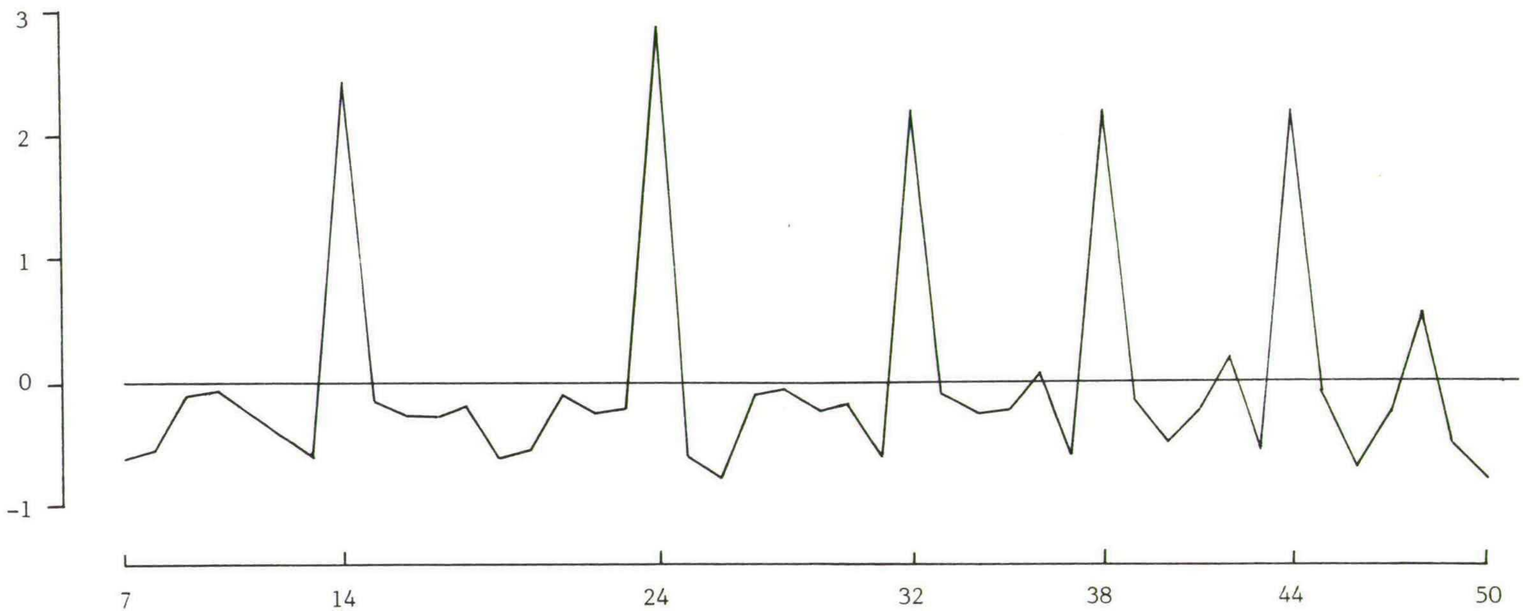
This enables us to construct a new time series, taking the first 6 observations of the original series as starting values. The result, which shall be referred to as the 'clear' series, is depicted in Figure 1.b. From this it appears that equation I describes a cycle with a periodicity of 12 periods. This is the dominant cycle, for it can be decomposed into three underlying cycles, of 3, 4 and 6 periods respectively. This is illustrated by Figure 3.

Looking at Figure 3 it is important to realize that a difference equation of recurrent cycles does not imply any specific cyclical pattern. The only thing that is really determined by it, is the periodicity of the cycle. Other features of the path, such as the amplitude, also depend on the starting values. This is not hard to see, for in the absence of shocks any recurrent cycle has to reproduce its initial conditions at the end of it. (Appendix 2 gives some examples of the effects of initial conditions.)

Of course, it is always possible to describe the inherent cyclical pattern of a recurrent cycle by the (isolated) effects of shocks. Table 2 contains some examples, constructed with the aid of equation I. Using this information it is easy to (re)construct the time series of Figure 1.a from the 'clear' series of Figure 1.b. It is also interesting to note that only if the shock is sustained for exactly the length of the dominant cycle (here 12 periods), this cycle will not be repeated. Else, if a shock of shorter duration is going on, the effects are lasting, consequently changing the shape of the series to which they are added.

Another important point to realize is that a difference equation of superimposed recurrent cycles may exhibit a very long dominant cycle.

Figure 2 Residuals



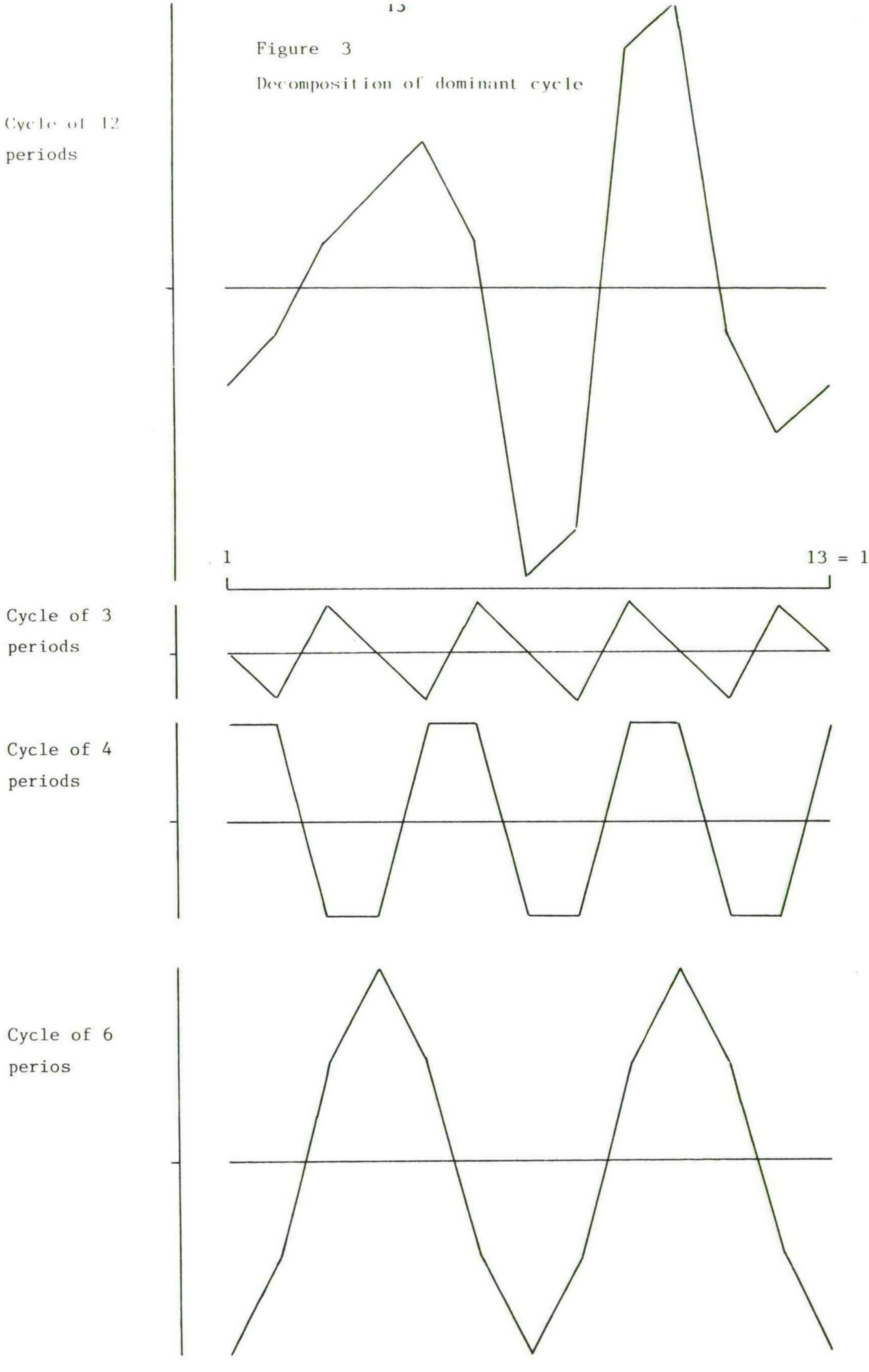


Figure 3
Decomposition of dominant cycle

The following example illustrates this:

$$\text{II } y_t = 0.382y_{t-1} - 2.382y_{t-2} + 0.764y_{t-3} - 2.382y_{t-4} + 0.382y_{t-5} - y_{t-6}$$

(Other examples have been put together in Appendix 3.)

At first sight this relationship differs much from equation I. However, actually they have two θ's in common, whereas the third θ is 108° in stead of 120°. Notwithstanding this small difference, equation II shows a dominant cycle, which is 5 times as long as that of example I: 60 in stead of 12 periods. This is due to the fact that the periodicity of the dominant cycle can only be observed as an integer, being the least common multiple of the periodicities of the underlying cycles. This has been illustrated by Figure 4. The upper half is a reproduction of Figure 1.b. It shows a recurrent cycle of 12 periods, which is clearly observable from period 10 onwards. The lower half of Figure 4 is based on equation II, starting from the same initial conditions as for equation I. (The corresponding figures have been reproduced in Appendix 4.)

Figure 4 demonstrates that the two time series coincide in the periods 55-60. Evidently, this is the time interval that both equations reproduce the initial conditions. However, of more importance is the coincidence of most peaks and throughs, thus showing the 'near' dominancy of the 12-period cycle in the case of equation II.

Regressing actual time series, it will be clear that the estimated θ's hardly ever shall appear as integers. Therefore, in empirical investigations, it is recommended to search for the periodicity of (what could be called) the 'nearly' dominant cycle. In most cases this will be possible, because the periodicities of the underlying cycles are estimated parameters, for which always can be tested whether they fit to an integer or not.

Table 2. Effects of shocks*

Shocks in period 1			Shocks in periods 1-6			Shocks in period 1-12		
Period	Effect	Cumulated	Period	Effect	Cumulated	Period	Effect	Cumulated
1	3	3	1	3	3	1	3	3
2	0	3	2	3	6	2	3	6
3	-6	-3	3	-3	3	3	-3	3
4	0	-3	4	-3	0	4	-3	0
5	6	3	5	3	3	5	3	3
6	0	3	6	3	6	6	3	6
7	-3	0	7	-3	3	7	0	6
8	0	0	8	-3	0	8	0	6
9	0	0	9	3	3	9	0	6
10	0	0	10	3	6	10	0	6
11	0	0	11	-3	3	11	0	6
12	0	0	12	-3	0	12	0	6
13	3	3	13	3	3	13	0	6
14	0	3	14	3	6	14	0	6
15	-6	-3	15	-3	3	15	0	6
16	0	-3	16	-3	0	16	0	6
17	6	3	17	3	3	17	0	6
18	0	3	18	3	6	18	0	6
19	-3	0	19	-3	3	19	0	6
20	0	0	20	-3	0	20	0	6
21	0	0	21	3	3	21	0	6
22	0	0	22	3	6	22	0	6
23	0	0	23	-3	3	23	0	6
24	0	0	24	-3	0	24	0	6

* Calculated with equation I; the value of a shock is 3 per period.

Figure 4.a Time series according to equation I

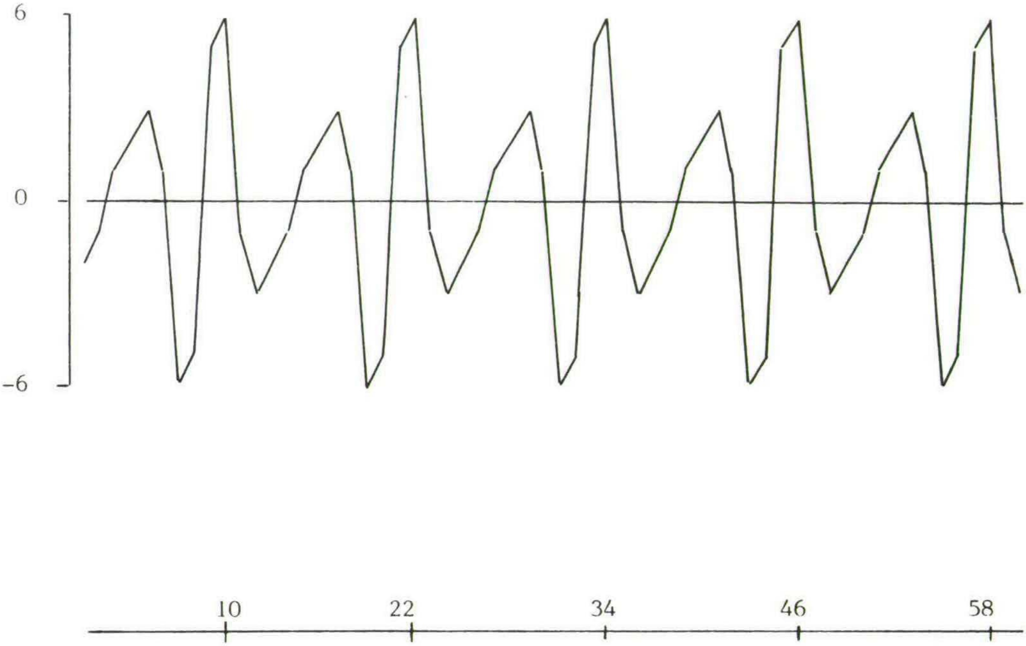
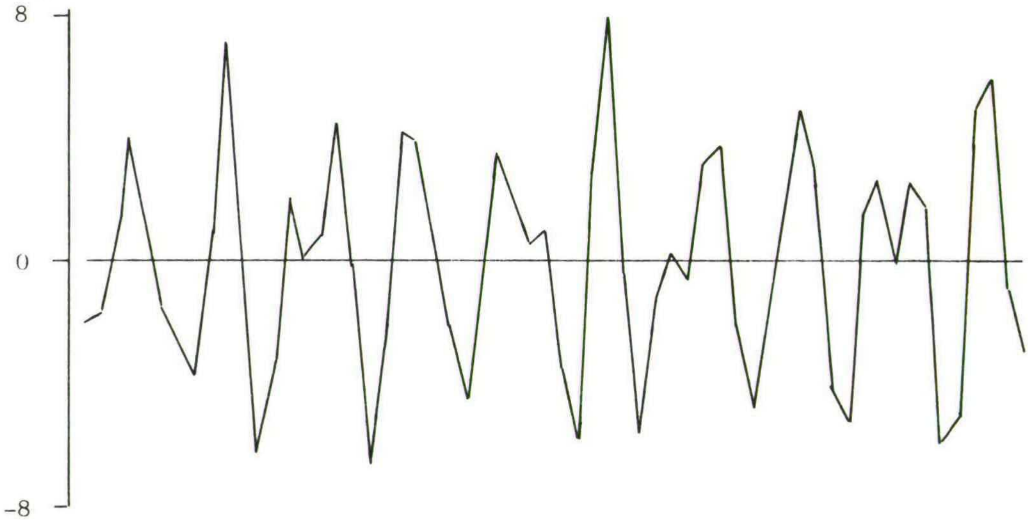


Figure 4.b Time series according to equation II



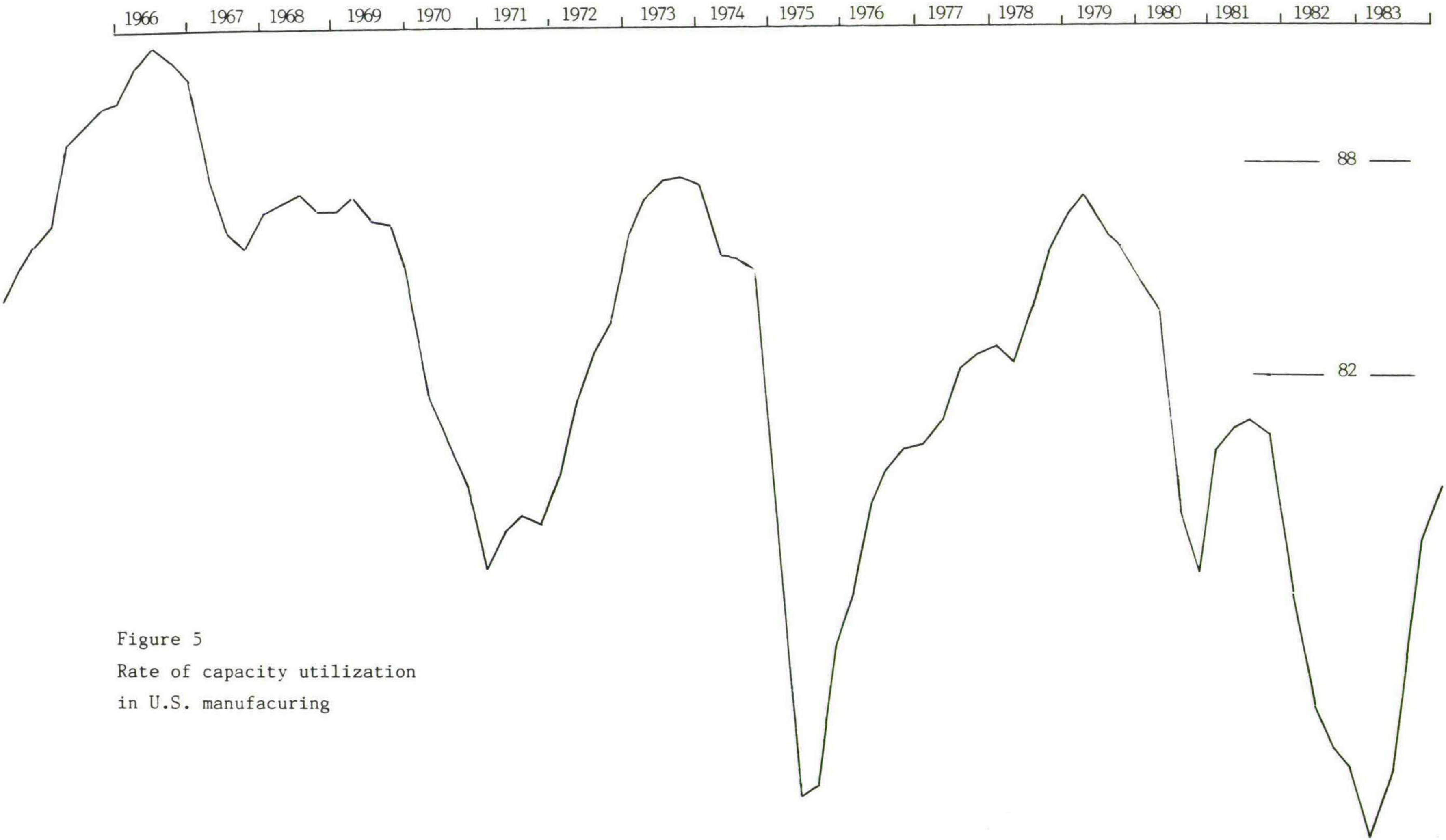


Figure 5
Rate of capacity utilization
in U.S. manufacuring

4. An empirical example

We now take up the challenge to employ the method presented above to empirical time series. For this we have chosen an example, which is known to be an important indicator of the state of the U.S. economy, namely the rate of capacity utilization in U.S. manufacturing. There is also another good reason to pay attention to this specific variable here, for in this case there is no need to distinguish between growth cycles and business cycles. Whereas growth cycles are fluctuations in economic indicators that are adjusted for their trends, business cycles are not. Thus the simple fact that the rate of capacity utilization shows no trend, makes it possible to concentrate on its short and medium-term fluctuations without correcting it, in some arbitrary way, for its long term movement, i.e. without detrending it.*)

The time series used here is drawn from OECD (1984) and involves quarterly material over the period 1964-1983. Figure 5 shows this series, which for completeness' sake is reproduced in Appendix 5. Given the well-known qualifications on aggregate capital utilization measures, this series seems to exhibit a recurrent cycle of medium-term length. This will be tested now.

Table 3 shows the difference equations that have been investigated. From this it may be concluded that the conditioned equations perform better than the corresponding unconditioned specifications, at least when comparing the t-ratio's and the coefficients of determination. It should be remarked that this result is not inherent to the specific time series under consideration. Applying the procedure on other time series shows the same pattern, which evidently is explained by the fact that additional information is introduced by imposing restrictions upon the estimated parameters.

*) As Zarnowitz (1985) points out, detrending time series may not be a good practice. Whereas growth cycles tend to be relatively symmetrical, business cycles show a strong asymmetry in expansions lasting longer than contractions.

Table 3. Regressions for the rate of capacity utilization in U.S. manufacturing*

1. $$y_t = \underset{(1.55)}{5.249} + \underset{(22.90)}{0.936}\, y_{t-1}$$

$R^2_{d.f.} = 78.0\%$

2. $$y_t = \underset{(2.92)}{8.545} + \underset{(14.87)}{1.442}\, y_{t-1} - \underset{(-5.60)}{0.546}\, y_{t-2}$$

$R^2_{d.f.} = 90.7\%$

3. $$y_t = \underset{(2.16)}{6.748} + \underset{(13.40)}{1.543}\, y_{t-1} - \underset{(-4.23)}{0.820}\, y_{t-2} + \underset{(1.64)}{0.195}\, y_{t-3}$$

$R^2_{d.f.} = 90.9\%$

4. $$y_t = \underset{(2.74)}{8.892} + \underset{(13.74)}{1.584}\, y_{t-1} - \underset{(-4.76)}{1.007}\, y_{t-2} + \underset{(2.63)}{0.561}\, y_{t-3} - \underset{(-2.05)}{0.247}\, y_{t-4}$$

$R^2_{d.f.} = 91.2\%$

5. $$y_t = \underset{(2.34)}{8.187} + \underset{(13.51)}{1.598}\, y_{t-1} - \underset{(-4.75)}{1.048}\, y_{t-2} + \underset{(2.62)}{0.663}\, y_{t-3} - \underset{(-1.67)}{0.370}\, y_{t-4} + \underset{(0.70)}{0.087}\, y_{t-5}$$

$R^2_{d.f.} = 91.3\%$

6. $$y_t = \underset{(2.24)}{8.442} + \underset{(13.12)}{1.606}\, y_{t-1} - \underset{(-4.63)}{1.069}\, y_{t-2} + \underset{(2.55)}{0.660}\, y_{t-3} - \underset{(-1.58)}{0.407}\, y_{t-4} + \underset{(0.61)}{0.142}\, y_{t-5} - \underset{(-0.28)}{0.036}\, y_{t-6}$$

$R^2_{d.f.} = 91.0\%$

$$7.\ y_t = \underset{(3.47)}{11.233} + \underset{(47.61)}{1.864}\, y_{t-1} - y_{t-2}$$

$R^2_{d.f.} = 96.7\%$ $\theta = 21.3°$

$$8.\ y_t = \underset{(4.04)}{15.238} + \underset{(13.66)}{1.696}\, y_{t-1} - \underset{(-6.72)}{1.575}\, y_{t-2} + \underset{(13.66)}{1.696}\, y_{t-3} - y_{t-4}$$

$R^2_{d.f.} = 95.5\%$ $\theta_1 = 16.5°$ $\theta_2 = 96.4°$

$$9.\ y_t = \underset{(3.70)}{17.441} + \underset{(12.74)}{1.657}\, y_{t-1} - \underset{(-5.41)}{1.398}\, y_{t-2} + \underset{(3.87)}{1.270}\, y_{t-3} - \underset{(-5.41)}{1.398}\, y_{t-4} + \underset{(12.74)}{1.657}\, y_{t-5} - y_{t-6}$$

$R^2_{d.f.} = 93.1\%$ $\theta_1 \approx 14.1°$ $\theta_2 = 126.1°$ $\theta_3 \approx 63.4°$

* Quarterly figures from OECD (1984).

This raises the question whether the right information is used by pinning the results into the preconceived direction. In the case of the fictive time series of Table 1 this was clearly justified. But now, from the unconditioned equations in Table 3, it appears that the expected restrictions do not announce themselves. This suggests that it is not allowed to conclude that the rate of capacity utilization in U.S. manufacturing exhibits a (dominant) self-sustaining cycle.

However, on closer investigation it is seen that we here are faced with the problem of having variables with a high degree of multicollinearity. (For the fictive time series of Table 1 this is not the case.) Consequently the estimates of the separate effects of the regressors are not reliable, thus preventing us from the conclusion that the time series under consideration exhibits a dying cycle, which has been kept alive with external shocks.

At this stage the problem is undecided. But there are additional criteria which give more support to the ultimate conclusion that the cycle in the rate of capacity utilization is self-sustaining in stead of dying. In the first place it is seen that the estimated parameters of the unconditioned equations all show the same sign as their conditioned counterparts. Further, moving up from the second to a higher order specification, the coefficient of y_{t-2} approaches (minus) one, being the restriction imposed upon equation (7). However, the most important indication arises after the addition of dummy variables, meant to catch the exogenous pressure on the rate of capacity utilization of the worldwide supply shocks of 1973-1974 and 1979-1980. From this it appears that the values of the estimated parameters of the unconditioned equations are very sensible for the addition of dummy variables, whereas the results for the conditioned relationships remain almost unchanged. This altogether leads to the conclusion that the rate of capacity utilization in U.S. manufacturing is adequately described by a self-sustaining cycle of approximately 4 years, which is inherent to equation (7).

Table 4. Rate of capacity utilization in U.S. manufacturing (1984-1989)*

1984-1	80.9	1985-1	88.3	1986-1	85.3	1987-1	77.4	1988-1	79.0	1989-1	87.2
1984-2	83.1	1985-2	88.7	1986-2	83.1	1987-2	76.6	1988-2	81.0	1989-2	88.3
1984-3	85.3	1985-3	88.3	1986-3	80.9	1987-3	76.6	1988-3	83.3	1989-3	88.7
1984-4	87.1	1985-4	87.1	1986-4	78.9	1987-4	77.5	1988-4	85.4	1989-4	88.2

* Forward calculation with equation (7) from Table 3.

It is an interesting experiment to use an autoregressive relationship for prediction purposes, certainly in the case that self-sustaining cycles are involved. Using equation (7), Table 4 gives the results of such an experiment. In this case the initial conditions originate from the upswing of 1983. Therefore, it is not surprising to see that the rate of capacity utilization moves through an expansion in 1984 to a peak in 1985 and then turns into a further contraction phase. The next lower turner point of the cycle is found in 1987, so that from thereupon the following expansion can come through. It goes without saying that this forward calculation is mechanistic, neglecting the influence of the recent supply shock.

5. Epilogue

The theory of (superimposed) recurrent cycles has always been a fascinating subject. In this respect, efforts to test the existence of a Kondratieff-cycle strike the imagination most. Sometimes these attempts incorporate additional (or preceding) findings on other cycles, such as the Juglar and the Kuznets, but on the whole they lack an integrating framework.

The method described above may provide us with a good starting point to develop such a framework. For, using difference equations conditioned for no dampening, it is not hard to detect long waves. This is illustrated by the following equation, which is based on a very long time series (1264-1954) for price indices of Southern England:*

$$y_t = \underset{(1.93)}{10.450} + \underset{(293.48)}{1.982}\, y_{t-1} - y_{t-2}$$

$$R^2_{d.f.} = 99.2\% \qquad \theta = 7.7^{\circ}$$

This equation shows that a recurrent cycle of approximately 47 years is inherent to the time series under consideration.

Imitating a single economic time series by the simple tool of an autoregressive relationship also stands at the core of the so called rational expectation revolution. In this literature it is a widely held view that (growth) cycles can be well described by stochastically disturbed difference equations of very low order, thus making the facts of life much more simple than they look.* Consequently it is not surprising that in the meantime this view has penetrated into many introductionary courses on economics.

An example of the latter is the textbook of Parkin (1984). There it is told that the fluctuations around trend in U.S. real income are well described by a second order difference equation, whereas the residuals are interpreted as purely random disturbances.

*) Source: Ramsey (1971)

*) See for instance Lucas (1977).

From a statistical point of view Parkin is right in stipulating that an (unconditioned) second order equation fits best to his detrended growth rates. But from the same point of view (See Appendix 6) it can also be concluded that a conditioned second order equation fits better, thus throwing doubt upon (Parkin's) empirical validation of the theory of dying cycles, which must be kept alive with random shocks.

Appendix 1. Series of the Figures 1, 2 and 3*

	(1)	(2)	(3)	(4)	(5)	(6)		(1)	(2)	(3)	(4)	(5)	(6)
1	-2	*	-2	0	2	-4	26	-4	-0.79	-1	-1	2	-2
2	-1	*	-1	-1	2	-2	27	1	-0.09	1	1	-2	2
3	1	*	1	1	-2	2	28	2	-0.01	2	0	-2	4
4	2	*	2	0	-2	4	29	3	-0.25	3	-1	2	2
5	3	*	3	-1	2	2	30	4	-0.17	1	1	2	-2
6	1	*	1	1	2	-2	31	-6	-0.63	-6	0	-2	-4
7	-6	-0.63	-6	0	-2	-4	32	-5	2.21	-5	-1	-2	-2
8	-5	-0.56	-5	-1	-2	-2	33	5	-0.09	5	1	2	2
9	5	-0.09	5	1	2	2	34	0	-0.25	6	0	2	4
10	6	-0.01	6	0	2	4	35	-1	-0.25	-1	-1	-2	2
11	-1	-0.25	-1	-1	-2	2	36	6	0.07	-3	1	-2	-2
12	-3	-0.40	-3	1	-2	-2	37	-2	-0.63	-2	0	2	-4
13	-2	-0.63	-2	0	2	-4	38	-4	2.21	-1	-1	2	-2
14	2	2.44	-1	-1	2	-2	39	1	-0.09	1	1	-2	2
15	1	-0.09	1	1	-2	2	40	-4	-0.48	2	0	-2	4
16	-4	-0.25	2	0	-2	4	41	3	-0.25	3	-1	2	2
17	3	-0.25	3	-1	2	2	42	10	0.30	1	1	2	-2
18	7	-0.17	1	1	2	-2	43	-6	-0.63	-6	0	-2	-4
19	-6	-0.63	-6	0	-2	-4	44	-5	2.21	-5	-1	-2	-2
20	-8	-0.56	-5	-1	-2	-2	45	5	-0.09	5	1	2	2
21	5	-0.09	5	1	2	2	46	-6	-0.71	6	0	2	4
22	6	-0.25	6	0	2	4	47	-1	-0.25	-1	-1	-2	2
23	-1	-0.25	-1	-1	-2	2	48	12	0.53	-3	1	-2	-2
24	0	2.83	-3	1	-2	-2	49	-2	-0.63	-2	0	2	-4
25	-2	-0.63	-2	0	2	-4	50	-7	-0.79	-1	-1	2	-2

* (1) Time series with irregular fluctuations (Figure 1.a)

(2) Residuals (Figure 2)

(3) Series with the dominant cycle of 12 periods (Figure 1.b)

(4) Series with the underlying cycle of 3 periods (Figure 3)

(5) Series with the underlying cycle of 4 periods (Figure 3)

(6) Series with the underlying cycle of 6 periods (Figure 3)

Appendix 2. Effects of initial conditions*

	Dominant cycle	Cycle of 3 periods	Cycle of 4 periods	Cycle of 6 periods
1	-18	3	-12	- 9
2	-13	-7	- 9	3
3	28	4	12	12
4	21	3	9	9
5	-22	-7	-12	- 3
6	-17	4	- 9	-12
7	6	3	12	- 9
8	5	-7	9	3
9	4	4	-12	12
10	3	3	- 9	9
11	2	-7	12	- 3
12	1	4	9	-12
1	2.5	-0.75	- 0.5	3.75
2	4	0.5	6.5	- 3
3	- 6	0.25	0.5	- 6.75
4	-11	-0.75	- 6.5	- 3.75
5	3	0.5	- 0.5	3
6	13.5	0.25	6.5	6.75
7	3.5	-0.75	0.5	3.75
8	- 9	0.5	- 6.5	- 3
9	- 7	0.25	- 0.5	- 6.75
10	2	-0.75	6.5	- 3.75
11	4	0.5	0.5	3
12	0.5	0.25	- 6.5	6.75
1	- 5	1	- 3	- 3
2	- 5	-2	- 3	0
3	7	1	3	3
4	7	1	3	3
5	- 5	-2	- 3	0
6	- 5	1	- 3	- 3
7	1	1	3	- 3
8	1	-2	3	0
9	1	1	- 3	3
10	1	1	- 3	3
11	1	-2	3	0
12	1	1	3	- 3
1	3	0	3	0
2	0	1.5	0	- 1.5
3	- 6	-1.5	- 3	- 1.5
4	0	0	0	0
5	6	1.5	3	1.5
6	0	-1.5	0	1.5
7	- 3	0	- 3	0
8	0	1.5	0	- 1.5
9	0	-1.5	3	- 1.5
10	0	0	0	0
11	0	1.5	- 3	1.5
12	0	-1.5	0	1.5

* Calculated with equation I. The 4th example is the decomposition of a shock in period 1.

Appendix 3. <u>Examples of superimposed undamped cycles</u>

ρ_1	ρ_2	ρ_3	a	c	e	θ_1	θ_2	θ_3	P_1	P_2	P_3	P_d
-1	2		0	0.5		90°	60°		4	6		12
-1.3473	2.3480		0.1736	0.5		80°	60°		4.5	6		18
-0.8308	2		0	0.4154		90°	65.4545°		4	5.5		44
-1.618	2		0	0.8090		90°	36°		4	10		20
-1.7125	2.1253		0.0383	0.8179		87.8049°	35.1220°		4.1	10.25		41
-1.6946	2.1239		0.0383	0.8090		87.8048°	36°		4.1	10		410
0	2	0	-0.5	0	0.5	120°	90°	60°	3	4	6	12
-0.3473	2	-0.3473	-0.5	0.1736	0.5	120°	80°	60°	3	4.5	6	18
-2.1462	2.3032	-1.3430	-0.5	0.7071	0.8660	120°	45°	30°	3	8	12	24
-1.2856	2	-1.2856	-0.5	0.5	0.6428	120°	60°	50°	3	6	7.2	36
-3.5202	6.0778	-7.0404	0	0.8090	0.9511	90°	36°	18°	4	10	20	20
-2.4142	4.4142	-4.8284	0	0.5	0.7071	90°	60°	45°	4	6	8	24
-3.5878	6.1962	-7.1756	0	0.8230	0.9709	90°	34.6154°	13.8462°	4	10.4	26	52
-3.6712	6.4734	-7.5874	0.0383	0.8090	0.9883	87.8049°	36°	8.7805°	4.1	10	41	410
-0.3820	2.3820	-0.7639	0.5	0	-0.3090	60°	90°	108°	6	4	3.33	60

Appendix 4. <u>Times series according to equations I and II</u>

Equation I

	Series			Series			Series	
- 5	-6		17	3	3	39	1	-2
- 4	-5		18	1	4	40	2	0
- 3	5		19	-6	-2	41	3	3
- 2	6		20	-5	-7	42	1	4
- 1	-1		21	5	-2	43	-6	-2
0	-3		22	6	4	44	-5	-7
1	-2	-2	23	-1	3	45	5	-2
2	-1	-3	24	-3	0	46	6	4
3	1	-2	25	-2	-2	47	-1	3
4	2	0	26	-1	-3	48	-3	0
5	3	3	27	1	-2	49	-2	-2
6	1	4	28	2	0	50	-1	-3
7	-6	-2	29	3	3	51	1	-2
8	-5	-7	30	1	4	52	2	0
9	5	-2	31	-6	-2	53	3	3
10	6	4	32	-5	-7	54	1	4
11	-1	3	33	5	-2	55	-6	-2
12	-3	0	34	6	4	56	-5	-7
13	-2	-2	35	-1	3	57	5	-2
14	-1	-3	36	-3	0	58	6	4
15	1	-2	37	-2	-2	59	-1	3
16	2	0	38	-1	-3	60	-3	0

Equation II

	Series			Series			Series	
- 5	-6		17	4.528	3.764	39	-0.528	-2.764
- 4	-5		18	-0.236	3.528	40	3.236	0.472
- 3	5		19	-6.764	-3.236	41	3.764	4.236
- 2	6		20	-2.528	-5.764	42	-2.236	2.000
- 1	-1		21	4.236	-1.528	43	-4.764	-2.764
0	-3		22	4.000	2.472	44	-1.764	-4.528
1	-2.000	-2.000	23	1.000	3.472	45	1.764	-2.764
2	-1.764	-3.764	24	-2.236	1.236	46	4.764	2.000
3	1.472	-2.292	25	-4.472	-3.236	47	3.000	5.000
4	4.000	1.708	26	-1.000	-4.236	48	-4.236	0.764
5	1.292	3.000	27	3.472	-0.764	49	-5.236	-4.472
6	-1.472	1.528	28	2.000	1.236	50	1.472	-3.000
7	-2.764	-1.236	29	0.528	1.764	51	2.708	-0.292
8	-3.764	-5.000	30	1.000	2.764	52	0.000	-0.292
9	1.000	-4.000	31	-3.528	-0.764	53	2.528	2.236
10	7.236	3.236	32	-5.764	-6.528	54	1.764	4.000
11	2.236	5.472	33	3.000	-3.528	55	-6.000	-2.000
12	-6.236	-0.764	34	8.000	4.472	56	-5.000	-7.000
13	-3.236	-4.000	35	-0.236	4.236	57	5.000	-2.000
14	2.236	-1.764	36	-5.472	-1.236	58	6.000	4.000
15	0.236	-1.528	37	-1.236	-2.472	59	-1.000	3.000
16	0.764	-0.764	38	0.236	-2.236	60	-3.000	0.000

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